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RELATION BETWEEN A GENERALIZED WIDEBAND SIGNAL AND ITS EQUIVALENT SHORT PULSE WHEN APPLIED TO ARRAY ANTENNAS

John B. Payne III

TECHNICAL REPORT NO. RADC-TR-66-270
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Rome Air Development Center
Research and Technology Division
Air Force Systems Command
Griffiss Air Force Base, New York

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John B. Payne III

AFLC, GAFB, N.Y., 28 Jun 66-117

FOREWORD

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ABSTRACT

The response of an array antenna to a short pulse of bandwidth B can be easily calculated and intuitively explained. However, when pulse compression having a bandwidth equal to this short CW pulse is used to obtain a longer pulse (higher average power) while retaining good range resolution, questions of system response and attainable resolution arise. By treating this problem in general terms, the following theorem covering array responses to a generalized wideband signal (pulse compression, etc.) is proven.

Given a generalized signal s(t), with bandwidth B, the response of a time invariant (during signal duration) array antenna to this signal as a function of look angle after element summation and resultant signal match filtering (filter response $s^*(t)$) will be identical to the response as a function of θ obtained from a short precise pulse g(t), whose shape is identical to the compressed pulse shape.

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INTRODUCTION

To understand the transient and propagation effects of wideband signals propagating across a phased array aperture for off-boresight directions, consider Figure 1. Here, an r-f pulse of width T, center frequency f_0, and bandwidth B (where $B = 1/T$) is impinging on the array interface phased to receive signals from an angle θ off boresight. The array is considered to be steered by wideband phasing networks K_n that are lossless and introduce correct phase shifts of ϕn, at f_0, in each element. Here, n denotes the element number. In a phased-steered array redundant shifts, ϕn, greater than 360 degrees are eliminated so that $n\phi$ does not exceed 2π radians. This is in general, correct for most phased arrays since the phase shift is usually introduced before summing by shifting the L. O. signal of a mixer placed in each element. Ferrite shifters have this same characteristic.

In a phase-steered system for off-boresight beam positions, as can be seen in Figure 1, the leading edge of the signal wavefront will arrive at element 0 first. The narrow pulse signal induced onto this element will be denoted by g(t). At a time τ later, the signal will be induced onto element 1. This signal is given by $g(t + \tau)$. The arrival of the signal at the nth element is delayed by $n\tau$ and given by $g(t + n\tau)$. The element-to-element signal delay τ is

$$\tau = \frac{d}{c} \sin \theta \qquad (1)$$

where c is the speed of light.

Each element signal is passed through the lossless phasing or time delay network K_n.

Upon combining each element signal in a linear summing network (the electrical length between each element and the summed output are considered equal), the resultant signal G(t) is given by the expression

$$G(t,\ N\tau) = g(t) + g(t + \tau) + g(t + 2\tau) + \ldots g(t + N\tau). \qquad (2)$$

Here, it is assumed that there are N + 1 elements.
Due to the element-to-element delay for $\theta > 0$, this output will appear as a stepped ramp until steady state is reached.

The summing networks output waveform for a six-element array will appear as shown in Figure 2. The energy contained in this output signal waveform is less than if the six pulses were in perfect time coincidence, as would be the case for $\theta = 0$.

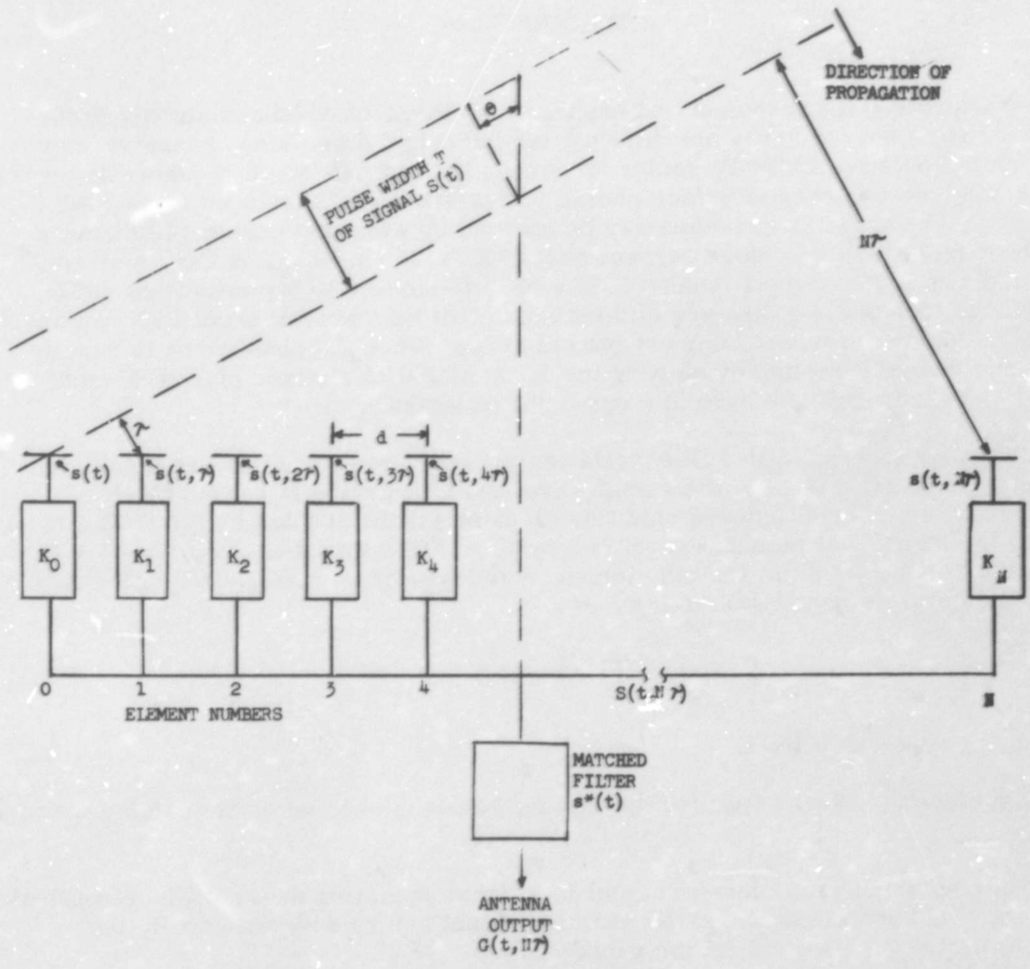

Figure 1. Array Antenna With Matched Filter After Element Summation

(Time coincidence for $\theta \neq 0$ can only be obtained by true time delays in each element)*. The further off-boresight the main beam is steered, the larger τ becomes and thus the more the waveform is smeared. This not only results in a decrease in signal energy, but also reduces the range resolution properties. In other words, the array acts like a filter whose bandwidth decreases as θ increases.

***For time delay steering equation (2) can be written**

$$G(t,\ N\tau) = g(t) + g(t + \tau - \tau_1) + g(t + 2\tau - \tau_2) + \ldots g(t + N\tau - \tau N)$$

Coincidence at the summing networks output occurs when

$$\tau = \tau_1,\ 2\tau = \tau_2,\ \ldots .\ N\tau = \tau N$$

Thus

$$G(t,\ N\tau) = Ng(t).$$

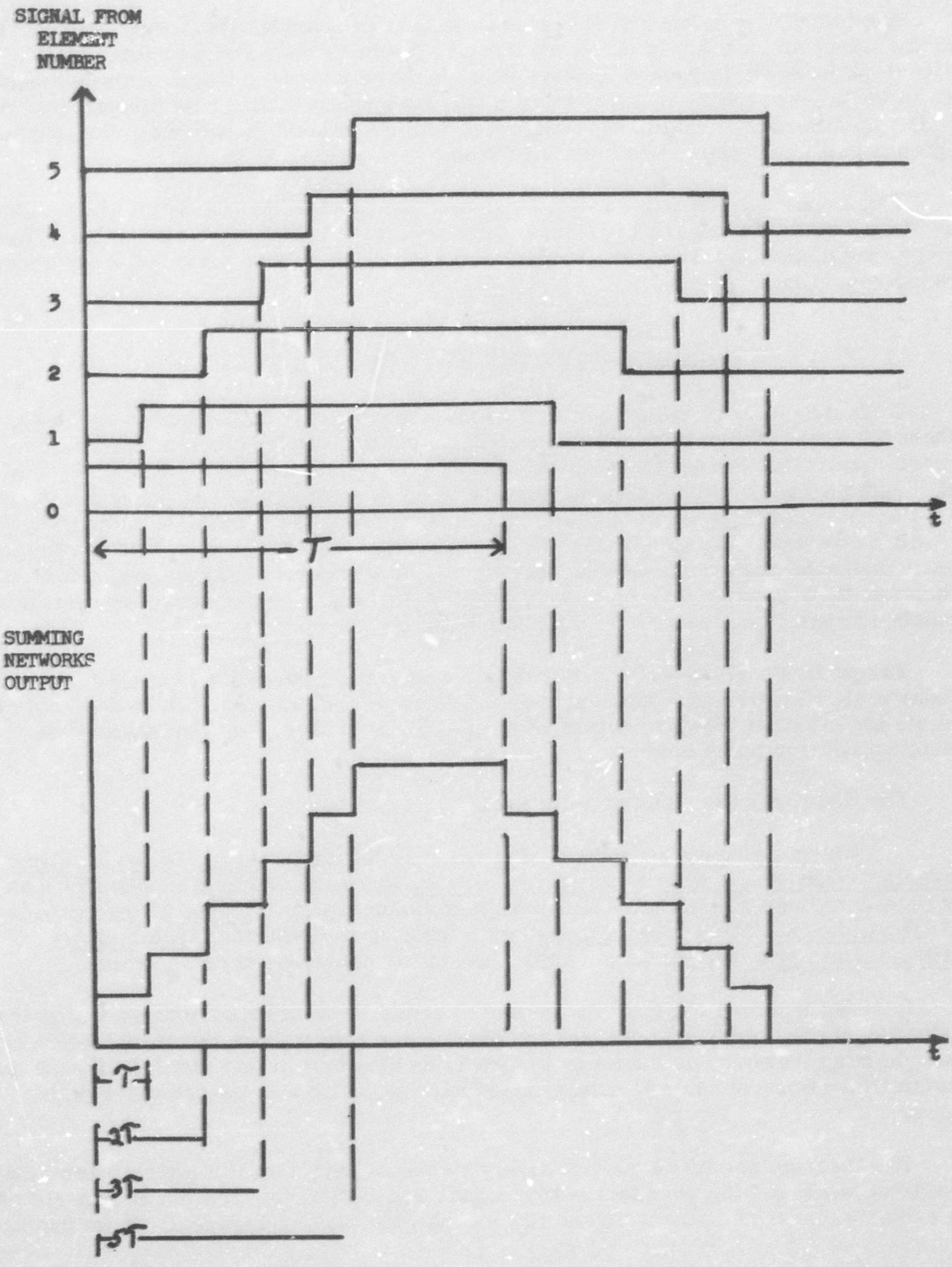

Figure 2. Propagation Effect of a Short Pulse on a Six-Element Array Antenna

Another way of looking at this energy loss is to consider the array's response to the signal spectrum. In a phase-steered system (as compared to time delay steering) the beam is phased to steer at a single carrier frequency. Power radiated at different frequencies in the spectrum will be steered to different directions. A wider spectrum of transmitted frequencies results in beam broadening (decreased antenna gain), and thus signal deterioration.

The longer the CW pulse width (narrower the bandwidth), the less effect smearing or beam broadening has on the total energy content of the array output pulse. The longer pulse also increases the system's average power for a given PRF but decreased range resolution.

SHORT PULSE EQUIVALENCY THEOREM

When pulse compression having a bandwidth approximately equal to the designed short CW pulse is incorporated in an array to obtain longer pulses as well as good range resolution, the question of system response, gain and resolution as a function of θ again arises. Often, a visual picture of an array response as described above for the short pulse case is not possible. System response could be obtained from specific waveforms by substitution in equation (2). However, the derivation for the many different compression techniques could become quite cumbersome. By treating equation (2) in general terms, a theorem covering array responses to a generalized wideband signal (pulse compression, etc.) can be proven.

Target information (primarily range resolution) is normally extracted from the radar's signal processing equipment in the form of a short pulse. It is desirable to relate the effect of the generalized wideband signal to that of its compressed or matched filtered pulse shape.

The theorem can be stated as follows:

Given a generalized signal s(t), with bandwidth B, the response of a time invariant (during signal duration) array antenna to this signal as a function of look angle after element summation and resultant signal match filtering (filter response $s^*(t)$) will be identical to the response as a function of θ obtained from a short precise pulse g(t), whose shape is identical to the compressed pulse shape.

In order to prove the theorem as stated above, it is necessary to show that the output signal $G(t, N\tau)$ from a filter matched to the generalized signal s(t) following the summing network, as shown in Figure 1, is identical to that obtained if only the match filtered waveform g(t) (compressed narrow pulse) was passed through the array.

The theorem can be shown intuitively by recognizing that the match filter is a linear network and the resultant output signal $G(t, N\tau)$ is just the linear superposition of each compressed element signal $g(t, n\tau)$ as indicated by Figure 3. If we can speak

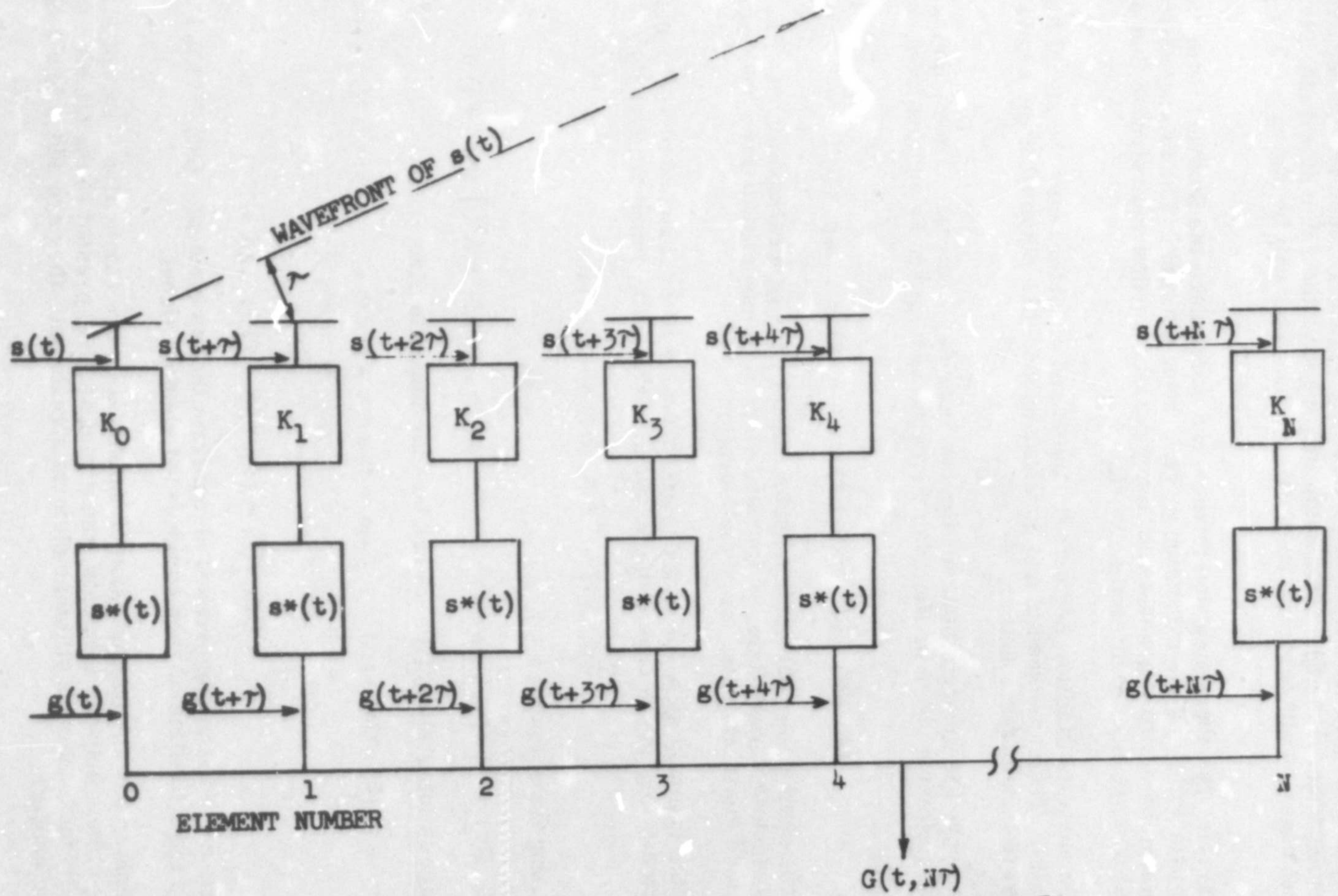

Figure 3. Array Antenna With Matched Filter in Each Element Before Summation

of each element signal $s(t, n\tau)$ as being compressed to $g(t, n\tau)$ before summation occurs, we could carry the compression network another step back and place it in space before the antenna elements indicated in Figure 4. Now we are receiving just the compressed pulse $g(t)$. Upon summing, we have the same signal $G(t, N\tau)$ as in the first case and our theorem has been proven. This will be shown mathematically below.

Carrying the above intuition further, we could place the matched filter in the transmitter following the expansion or generating network. The two networks can be replaced by a single generator which generates a pulse equivalent to that of the original compressed or filtered network.

The analysis to follow neglects the noise and considers only the signal for simplicity. The same results and conclusions would be obtained if the signal and noise were considered together.

To prove the theorem mathematically, consider Figure 1. The summing networks output for either a phase or time delay steered system can be written from equation (2) as:

$$S(t, N\tau) = s(t) + s(t + \tau) + s(t + 2\tau) + \ldots s(t + N\tau) \tag{3}$$

where the short pulse $g(t)$ has been substituted by the generalized signal $s(t)$. Here, the time invariant restriction is imposed since it is assumed that the phasing network, K_n, is held constant during the signal duration.

If this summed signal is now passed through a filter matched to $s(t)$ (i.e., compression network in the case of pulse compression), the output is given by

$$G(t, N\tau) = \int S(\alpha, N\tau)\, s^*(\alpha + t)\, d\alpha. \tag{4}$$

Substituting from equation (3)

$$G(t, N\tau) = \int \left[s(\alpha) + s(\alpha + \tau) + s(\alpha + 2\tau) + \ldots s(\alpha + N\tau)\right] s^*(\alpha + t)\, d\alpha \tag{5}$$

If $s(t)$ is integrable, then (5) can be rearranged to give (6)

$$G(t, N\tau) = \int s(\alpha)\, s^*(\alpha + t)\, d\alpha + \int s(\alpha + \tau)\, s^*(\alpha + t)\, d\alpha + \ldots \int s(\alpha + N\tau)\, s^*(\alpha + t)\, d\alpha$$

or

$$G(t, N\tau) = g(t) + g(t + \tau) + \ldots g(t + N\tau) \tag{7}$$

where $g(t + n\tau)$ is the compressed signal from the nth element. $G(t, N\tau)$ is thus seen to be the superposition of each compressed element signal.

Consider now the compression of the signal before summation. Except for mutual coupling effects between elements, the signal $s(t, n\tau)$ present at the nth element is essentially independent of all other element signals until they are added in a common summing network.

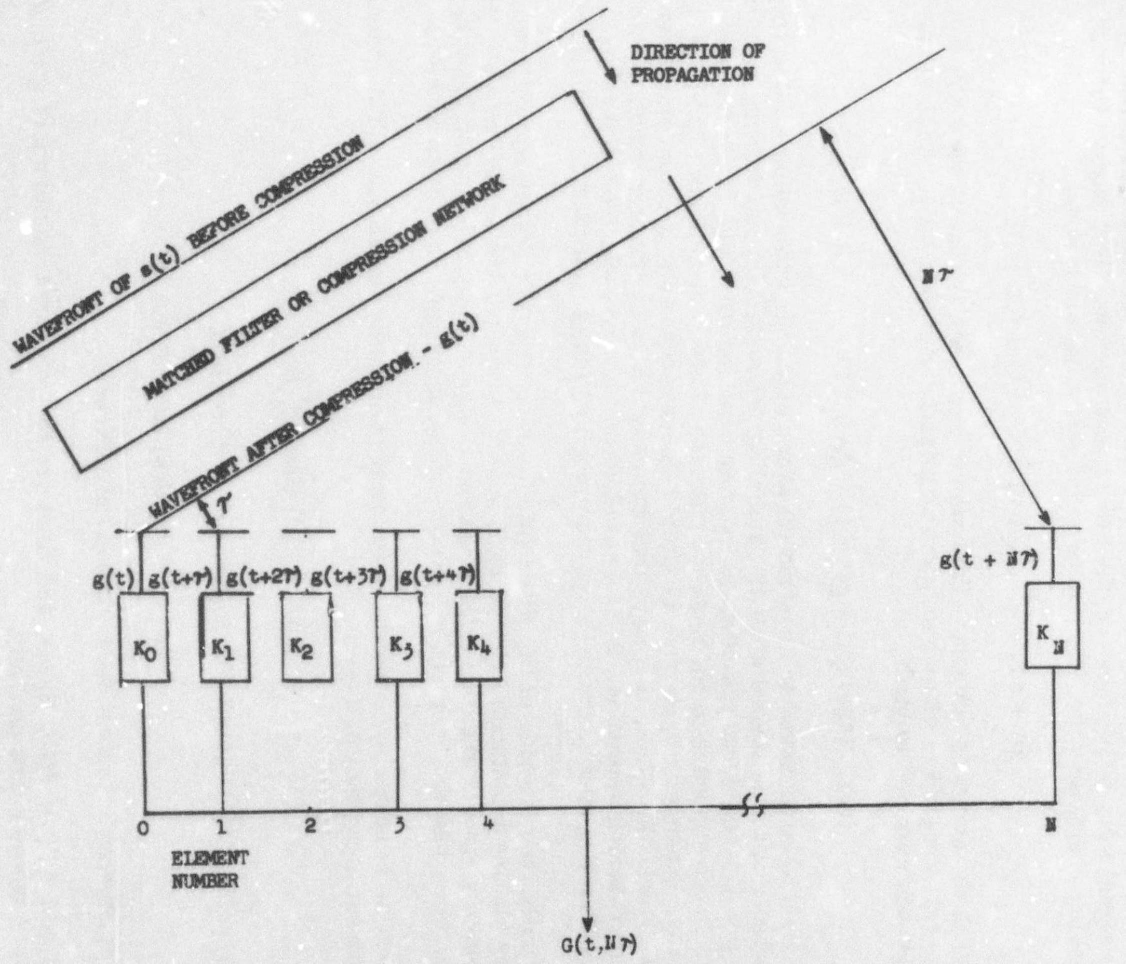

Figure 4. Array Antenna With Matched Filter Located In Space

We can operate on any one element signal in whatever way we wish before summing. In effect, this is what we do in array systems when we adjust the phasing or delay of each element channel to obtain coherent signal summation.

Let us place a matched filter in each antenna element channel as shown in Figure 3. The signal $s(t + n\tau)$ induced on the nth antenna element first passes through a filter having response $s^*(t)$. Its output to the summing network is:

$$g(t + n\tau) = \int s(\alpha,\ n\tau)\, s^*(\alpha + t)\, d\alpha. \tag{8}$$

$g(t + n\tau)$ is the compressed pulse shape whose bandwidth is that of $s(t + n\tau)$ (actually with spectral weighting, $g(t)$'s bandwidth is slightly less than $s(t)$). Each element signal is then summed to yield

$$G(t,\ N\tau) = g(t) + g(t + \tau) + g(t + 2\tau) + \ldots g(t + N\tau). \tag{9}$$

This is seen to be identical to equation (7) which gives the expression for the output signal when a filter is placed after the summing network. By substituting equation (8) into (9), we can obtain the results given by equation (6). The integrals in both cases yield the response of the matched filter to a different element signal. The final array response in each case is simply the superposition of each compressed element signal. In effect, this says that the convolution operation, which is just the match filtering operation, is both commutative and associative, so that for linear, time stationary systems, the order of the match filtering operation is immaterial.

Actually, the location of the filter (in either receiver or transmitter) is unimportant. By placing a matched filter at each antenna element, the array response will be the same as though the generalized wideband signal $s(t)$ incident upon the array interface were just the compressed narrow pulse signal $g(t)$, as shown in Figure 4.

That this is true can be shown as follows. Consider Figure 5(a). The signal fed to the summing network is

$$G'(\omega) = S_1(\omega)\, H(\omega)\, S_1^*(\omega) \tag{10}$$

where $S_1(\omega) = G''(\omega)\ S(\omega)$ from Figure 5(b). Therefore,

$$G'(\omega) = G''(\omega)\, S(\omega)\, H(\omega)\, S_1^*(\omega) \tag{11}$$

$H(\omega)$ is considered constant over the band of interest.

Here, $G'(\omega)$, $G''(\omega)$, $S(\omega)$, $S_1(\omega)$, and $H(\omega)$ are the Fourier transforms of $g'(t)$, $g''(t)$, $s(t)$, $s_1(t)$, and $h(t)$, respectively.

If the matched filter were assumed located in space (Figure 4) so that the signal induced on the antenna was

$$S(\omega) = G''(\omega)\, S(\omega)\, S^*(\omega)\, H(\omega), \tag{12}$$

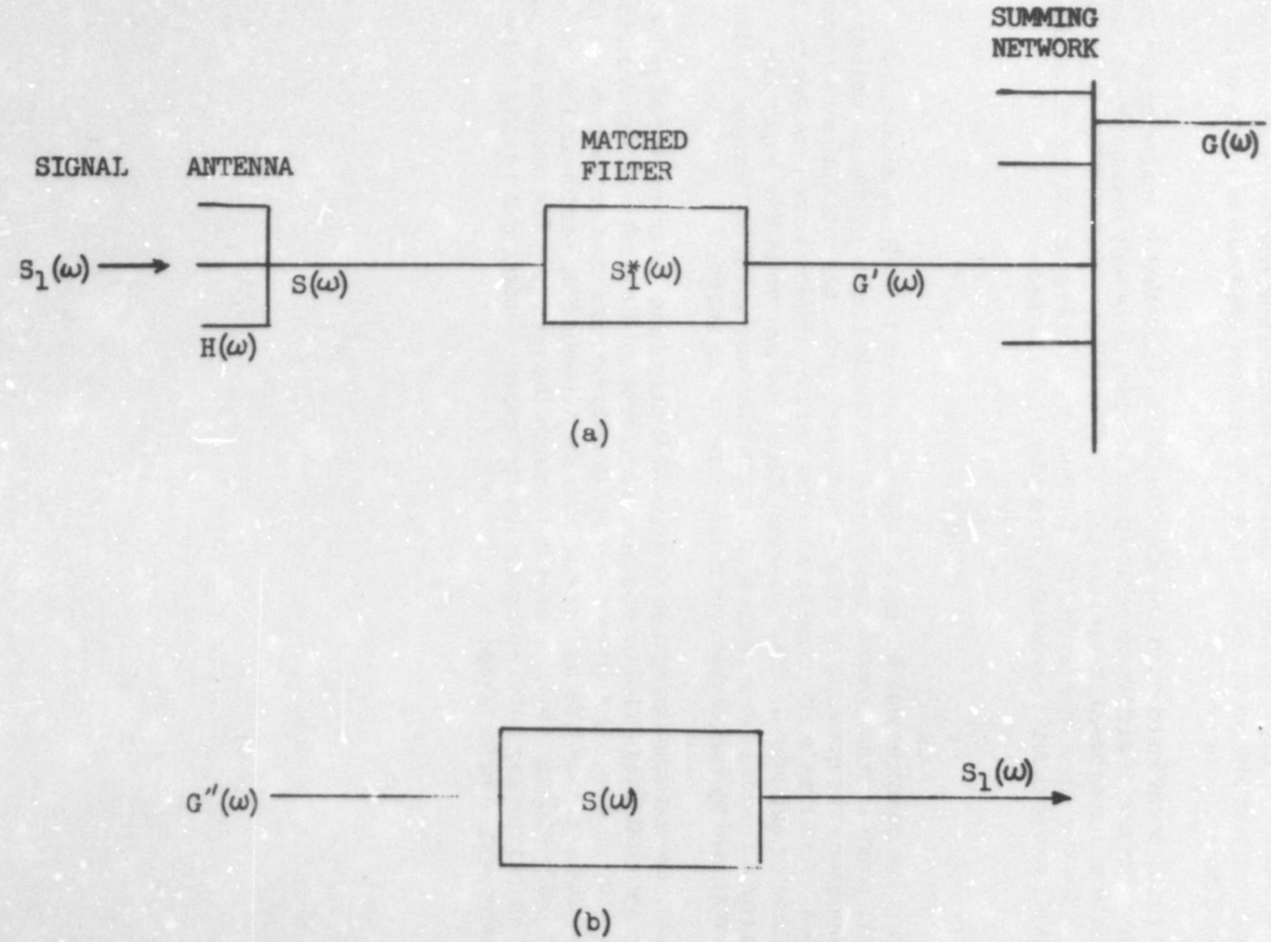

Figure 5. (a) Matched Filter Per Element of Array
(b) Expansion of Generating Network

the antenna output to the summing network then becomes

$$G'(\omega) = G''(\omega)\, S(\omega)\, S^*(\omega)\, H(\omega) \tag{13}$$

If $H(\omega)$ is constant over the band of interest and each function is well behaved and integrable, then equations (11) and (13) are identical and our statement is shown to be correct. That is, on an element basis it is unimportant where the compression takes place; we can always speak of the array element response in terms of the compressed pulse.

This proof could be carried one step further to show that in analyzing the complete round trip response of arrays to generalized signals, it is only necessary to consider the response of the over-all system to the compressed or matched filter pulse shape. One way of looking upon this is that the transmitter process is just the reciprocal of the receiving process and therefore has an identical response.

CONCLUSION

When pulse compression having a bandwidth approximately equal to the desired short C.W. pulse is incorporated in an array to obtain longer pulses as well as good range resolution, the question of system response, gain, and resolution as a function of θ arises. An array's response to an expanded or complex waveform can not always be visualized or calculated. The theorem stated and proven in this paper has shown that an array's response to an expanded or complex waveform is identical to the response obtained by the waveforms compressed pulse shape.

That is, the antennas response is dependent only upon the compressed pulse shape and not on the expanded or complex waveform used to transmit the desired energy or information. If a given short pulse is to be extracted from a radars signal processing section, it is unimportant whether linear FM, nonlinear FM, step frequency pulse coding, etc., is used to transmit the energy. The antenna response will be identical for any pulse compression technique, assuming it results in the desired compressed pulse shape.

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